Technical Comments

Comments on "Dynamics of a Spacecraft during Extension of Flexible Appendages"

M.S. Jankovic*
Dome Petroleum Ltd., Clagary, Canada

IT appears that the equations of motion derived in this paper do not describe the extension of flexible appendages, simply because the author treats the appendage's length as a constant parameter. He failed to recognize that the functions $E_n(\eta)$ for example, in Eqs. (4) and (8), are functions of space and *time also*, because $\eta = x/\ell(t)$ $[\eta = y/\ell(t)]$ is a function of time. Consequently, the equations of motion (12-15) do not contain the time derivatives $\dot{E}$, $\ddot{E}$, and $\dot{E}_\eta$ $[E \equiv E_n(\eta)]$, which must be included.

It is reasonable to expect that for a sufficiently small deployment velocity v (for example, $v/\Omega\ell \ll 1$, where Ω is the largest natural frequency of a fully deployed appendage at instantaneous length l at the fixed time), the elastic deformation of an appendage may be sought in terms of the series

$$U_i = \sum_{n=1}^{\infty} T_{in}(t) E_n(\eta) \tag{4}$$

where, for example, the functions $E_n(\eta)$ are defined by

$$\frac{d^4 E_n(\eta)}{d\eta^4} - \lambda_n E_n(\eta) = 0 \tag{5}$$

for the boom-type appendage.

It must be recognized that in general E_n is a nonlinear *function of time* as well as space and other time-dependent variables characterizing the appendage. Also λ_n in Eq. (5) is a *function of time*, too. However, the analysis in the paper implies that E_n and λ_n are *not* time functions! The proper equations of motions for the deployment of boom-type appendages are given in Ref. 1.

One way to solve this problem is to assume that the shape function for the boom-type appendage is of the form

$$E_n(\eta) = C_1 ch(\alpha\eta) + C_2 sh(\alpha\eta) + C_3\cos(\alpha\eta) + C_4\sin(\alpha\eta), \; [\alpha = \alpha(\lambda)] \tag{I}$$

which is the solution of Eq. (5), and then substitute it into the boundary conditions:

$$E_n(\eta) = \frac{dE_n(\eta)}{d\eta} = 0, \qquad \eta = 0$$
$$\frac{d^2E_n(\eta)}{d\eta^2} = \frac{d^3E_n(\eta)}{d\eta^3} = 0, \qquad \eta = 1 \tag{6}$$

which would lead to a nonlinear eigenvalue problem

$$\underline{A}(\lambda)\underline{c}(t) = \underline{0} \tag{II}$$

where underbar denotes matrices of appropriate order,

$$\underline{c}(t) = (C_1 C_2 C_3 C_4)^T$$

is the eigenvector, and $\lambda \equiv \lambda_n$ is the frequency.

Equation (II) has nontrivial solutions for

$$\det \underline{A}(\lambda) = 0 \tag{III}$$

which is a nonlinear frequency equation that must be solved numerically for λ at each instant of time corresponding to the instantaneous lengths ℓ. The frequencies λ are not natural frequencies for there are no natural frequencies for the appendage whose length changes with time (deployment case); therefore they could be called quasifrequencies.

The eigenvector $\underline{c}$ is normalized using

$$\int_0^1 E_n(\eta) E_m(\eta) d\eta = \delta_{n,m} \tag{7}$$

resulting in

$$\underline{c}^T \underline{P}(\lambda) \underline{c} = 1 \tag{IV}$$

where $\underline{P}(\lambda)$ is a function of λ.

Now from Eqs. (II) and (IV) time derivatives $\dot{\lambda}$, $\ddot{\lambda}$, $\dot{\underline{c}}$, and $\ddot{\underline{c}}$ can be analytically calculated, which are then used to determine $\dot{E}$, $\dot{E}_\eta$, and $\ddot{E}$ which must enter Eqs. (12-15).

This type of analysis is presented for a general case of deployment of flexible appendages in Ref. 2.

References

[1]Hughes, P.C., "Deployment Dynamics of the Communication Technology Satellite," presented at ESRO Symposium on Non-Rigid Vehicle Dynamics and Control, Frascatti, Italy, May 1976.

[2]Jankovic, M.S., "Deployment Dynamics of Flexible Spacecraft," Ph.D. Thesis, University of Toronto, Institute for Aerospace Studies, 1979.

Submitted April 25, 1983; revision received July 15, 1983.

*Senior Project Engineer. Member AIAA.

Reply by Author to M.S. Jankovic

Kazuo Tsuchiya*
Mitsubishi Electric Corporation
Amagasaki, Japan

THROUGHOUT the paper, Ref. 1, under the basic assumption that "the appendages are supposed to be extended very slowly," the adiabatic approximation together with a modal analysis has been employed: The basic equations, (12-15) in Ref. 1 are derived by neglecting small quantities of the second order in v, the deployment velocity of the appendages. Here, note that these equations correspond to Eqs. (24) and (25) in Ref. 2 by omitting terms above the first order in v. Equations (25), which are derived by the multiple scales method, are correct as far as the term in v. Hence, the whole of the analysis in Ref. 1 is based on the expansion of the motion of the system in terms of the deployment velocity v, retaining only the first-order terms. Such analysis correctly takes into account the first-order effect of the extension of the flexible appendages and is entirely legitimate when the deployment velocity is sufficiently small.

References

[1]Tsuchiya, K., "Dynamics of a Spacecraft during Extension of Flexible Appendages," *Journal of Guidance, Control, and Dynamics*, Vol. 6, No. 2, March-April 1983, pp. 100-103.

[2]Hughes, P.C., "Deployment Dynamics of a Communication Technology Satellite," ESA SP-117, July 1976, pp. 225-340.

Received July 25, 1983.

*Senior Research Scientist, Central Research Laboratory.